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Structural Design Space Exploration Using Principal Component Analysis

Design space exploration (DSE) is the process whereby a designer seeks to understand some results across a set of design variations. Structural DSE of turbomachinery compressor blades is often challenging because the large number of design variables make it difficult to learn the effect that each variable has upon the stress contours. Principal component analysis (PCA) of the stress contours is used as a way to understand how the stress contours change over the design space. Two methods are introduced to address the challenge of understanding how the stress changes over a large number of variables. First, a two-point correlation is applied to relate the design variables to the scores of each principal component. Second, a coupling of the stress and coordinate location of each node in PCA is developed which also indicates how the stress variations relate to geometric variations. These provide insight to how design variables influence the stress. It is shown how these methods use PCA as DSE tools to better explore the structural design space of compressor blades. Better DSE can improve compressor blades and the computational cost needed for their design. [DOI: 10.1115/1.4047428]

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1 Introduction

Design space exploration (DSE) is the study or analysis of designs defined by a set of variables which are permitted to vary. When done well, it can provide powerful insight into the influence of design variables on the local and global results. Optimization is a form of DSE which seeks to find a maximum or minimum objective in a design space subject to constraints. DSE is a broader concept which seeks to understand and explore the design space. It helps find optimums, understand sensitivities, and explore new design concepts. This form of exploration is useful for many types of design problems. One application is the multidisciplinary design of turbomachinery compressor blades. A geometry optimized aerodynamically may not be structurally feasible. This requires structural DSE around a proposed aerodynamic design to find a structurally feasible design.

A challenge of exploring the structural design space of compressor blades is knowing which design variables to vary, and by how much, to satisfy structural criteria. Compressor blade structural design often involves many iterations of trying different design variations, based on a proposed aerodynamic geometry, to find one which is structurally feasible. The structural and aerodynamic groups iterate together with each group also performing sub-iterations. This design method has a twofold problem which limits the ability to find quality, feasible designs. First, the long solution times required for high fidelity simulations limit the number of designs which can be explored. Second, a lack of understanding about how design variables influence compressor blade structural results obscures a designer's knowledge of which design variables to vary and which regions of the design space to explore. This may lead to a larger percentage of simulations being wasted on designs which do not improve the compressor blade design objectives. Previous research studied improving the

DSE speed for compressor blades by using surrogates to emulate the blade stress [1,2]. The present research focuses on methods to understand the relationship between design variables and structural results to improve DSE. This will help designers understand which design variables and regions of the design space should be explored.

This research uses principal component analysis (PCA) to develop methods which improve an understanding of compressor blade structural design spaces. PCA is also known in the field of turbomachinery and computational fluid dynamics (CFD) as proper orthogonal decomposition (POD) [3–5]. It provides the orthogonal variations in the structural data and scores which represent the magnitude of those variations for each sample. The orthogonality of the PCs means that the variations are the most efficient set possible from the given data. While this paper uses PCA, other latent or spectral methods may also be used if they provide a set of orthogonal variations and the magnitude of each variation for each sample. Other such acceptable methods include exploratory factor analysis or least squares regression [6,7]. Latent methods which would not work include hidden Markov models, Isomap, or other non-linear techniques because they do not provide orthogonal vectors which represent the variations [8]. PCA was selected due to its simplicity and given that the transformation preserves as much of the variability as possible within the first few PCs [9]. For DSE of compressor blades, the samples are design variations from a baseline compressor blade and the data are structural results from finite element analysis (FEA). Two methods are used to understand the relationship between design variables and PCs for high-dimensional spaces. The first method applies a two-point correlation between design variables and principal component (PC) scores. The second method was developed in this research from coupling the node locations and node stresses in a single PCA. These methods are applied to DSE of the von Mises stress on blade-alone finite element models of the transonic Purdue compressor [10]. This compressor blade was developed to study transonic aerodynamic effects but has been the subject of many academic studies, including structural PCA analysis [11]. The nominal design has a height of about 2.0 in, a mean chord of about 1.9 in, and a mean thickness to chord ratio is about 8%.

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The paper proceeds with a background on DSE and PCA of result fields. Then, the methods for PCA and the two-point correlation are described. Following this, the results of the PCA, two-point correlations, and stress and geometry coupling are shown. The results show PCs of the compressor blade across various design spaces and how the stated methods relate the variables to the PCs. The discussion then provides an example to demonstrate how to use these methods to improve compressor blade DSE.

2 Background

The background covers deficiencies to common practices of compressor blade DSE, the basics of PCA, and methods which have been used to better understand turbomachinery with PCA.

2.1 Design Exploration. Design space exploration of compressor blades is a complicated and computationally expensive process. It is common to start from a baseline design and rely on experience to adjust design variables until a desired result is obtained. For the structural design of compressor blades, each iteration requires FEA with possibly millions of nodes [12]. The long simulation times and computational expense which accompany these high fidelity models limit the number of designs which may be tested. Research has shown that during design exploration, longer times to reach an acceptable design lead to worse final designs [13–15]. This means that the simulation times should be as low as possible and that the designs which are tested should be chosen well. Knowing which designs to test can be challenging without an understanding of how each design variable influences the structural results. While engineering experience can help with this, that understanding is limited in unfamiliar or complex design spaces.

More advanced design exploration methods use a design of experiments (DOEs) to choose which designs to analyze. A DOE, also known as a sampling plan, seeks to select a set of designs which are, in some sense, evenly distributed through the design space [16]. Analysis from the selected samples can help understand global trends across the design space. Borer and Moore [17] and Huang [18] performed DSE by using a Latin hypercube DOE to understand how performance was related to specific propeller and aerospace engine design characteristics. After selecting and analyzing the samples, they plotted their performance parameters with their design parameters to understand the global trends and relationships in propeller and engine design variables. They were only exploring results based on single values. Many have also used DOEs and fit surrogate models to emulate the response fields of structural parts [1,2,19,20]. Their methods allowed for real-time design queries but lacked the ability to relate the design variables to the global response due to the response being represented by fields instead of single values.

Table 1 Set of possible design variables used for this research

Design variable	Symbol	Range
Height	H	$\pm 10\%$ of Nom. H
Ave. radius	R_o	$\pm 10\%$ of Nom. R_o
Root chord	C_R	$\pm 10\%$ of Nom. C_R
Tip chord	C_T	$\pm 10\%$ of Nom. C_T
Sweep	S	$\pm 10\%$ of Nom. C_T
Lean	L	$\pm 10\%$ of Nom. C_T
Angle	α	± 20 deg of Nom. N
Rotation speed	N	$\pm 10\%$ of Nom. N
Ave. pressure	P	$\pm 10\%$ of Nom. P

Note: The ranges for these variables were based on the relationship to a nominal value of the geometry.

2.2 Principal Component Analysis. Principal component analysis is a spectral method which creates a reduced-order model from data with a large number of dimensions while retaining as much information as possible [9,21]. It finds the orthogonal variations, or PCs, which explain the most variation in the data. Because they are orthogonal, each PC is independent from one another. By using only a few PCs which explain the most variation, the data can be more quickly analyzed and understood. Originally developed for statistics, it is used in other fields, often by different names and with slight variations. Despite these differences, the governing principles are the same. This research followed the basic method presented by Jolliffe and Cadima [9]. PCA is used in aerospace engineering for many reasons, including real-time analysis, data size reduction, control of the tradeoff between error and computation cost, and improved understanding of characteristics in the data [22]. Hajikolaie and Wang used PCA to reduce high-dimensional design spaces to design spaces of lower dimensions [23]. They found that by using just a few PCs, they could reconstruct the results from the original high-dimensional space with less than 5% relative average absolute error. This low error provides confidence that just the first few PCs indicate meaningful variations through the design space.

Principal component analysis has been applied to better understand and explore design spaces based on flow fields, heat transfer, and geometric tolerances of compressor blades, turbine blades, and other parts. [3,24,25]. Blanc et al. used PCA to reduce the set of nodal temperatures on a turbine blade finite element model to a small set of PCs [24]. These represented the orthogonal variations of the temperature field on the blade. The full temperature field was a linear combination of the PCs with their respective scores. The PCs provided a better understanding of the characteristics of this temperature field. Spencer et al. used PCs to understand unique characteristics about flow fields [25]. They were able to better understand specific flow characteristics and relate them to the simulation inlet conditions. Brown et al. used PCA to get geometric variations in as-manufactured compressor blades and used the scores to emulate blade mode shapes and frequencies [26]. Using the PC scores as inputs to the model allowed them to predict how changes in geometric variation affected the frequencies and modes. Yan and Ballu compared methods to understand the spatial variation of parts [21]. They found that among these methods, PCA performed the best at displaying and understanding the spatial variations of manufacturing defects. These studies used PCA as an analysis tool to better understand flow and temperature characteristics. The present research shows how similar PCA methods may be used as a DSE tool when coupled with methods which relate the design variables to PCs.

3 Method

Principal component analysis was used to develop methods to understand variations in the von Mises stress of compressor blade design spaces. This allowed global trends to be discovered when exploring result fields. The tested design spaces were made up of various combinations of the design variables shown in Table 1 for the transonic Purdue compressor blade. The table is made up of nine design variables in which the first and second columns have the names and symbols, respectively, of the design variables. The third column gives the range that each variable may deviate from its nominal value. Methods similar to Blanc et al., as described in the background, were applied to reduce the nodal results for sampled designs into the PCs [24]. These nodal results, which together represented the spatial distribution of von Mises stress in the samples, were reduced into PCs which described variations of spatial stress distribution among those samples. The spatial distributions describe how the stress varies over the surface of the blade. The PCs describe how that spatial distribution varies across the design space. This research, however, coupled nodal von Mises stress and nodal coordinates together in PCA. These PCs

represented the orthogonal variations which coupled stress and geometry variations present among the samples. The orthogonality of the PCs means that variations are independent of each other such that a change in the score of one PC does not necessitate the change in the score of another PC. This property is helpful when exploring the design space and isolating the effect of design variables upon the stress of the blade. The full spatial stress field and geometry for a sample in the design space could be reconstructed by a linear combination of the PCs.

The general workflow to create the data sets and model is shown in Fig. 1 where rectangles represent numerical processes and cylinders represent data. To train the model, data sets for the stresses and coordinates of each node on the finite element model were obtained for a number of design variations from the following steps. The first step was to choose which design space to explore and then select samples from that design space for simulation. This required selecting the design variables and creating a design of experiments specifying the designs to sample. Latin hyper-sampling with a maximin criterion was used to select samples spread across the design space. Then, a finite element model was mesh morphed to match the designs. This mesh morph was performed to keep the nodes in the same location, relative to the geometry, among all of the samples, which was necessary so that the stress at each point on the part could be recorded for each sample. The finite element model of the transonic Purdue compressor blade used in this study had 25,000 nodes with a fixed constraint at the blade root, a uniform pressure load, P , applied to the pressure surface, and a

rotational load, N . Next, FEA was used to solve for the stresses of the finite element model. Upon finishing FEA, the stress and location of each node were saved into a data set. The data sets used in this research contained stresses and coordinates for 25,000 nodes and up to 100 samples. The stresses had values up to 72 ksi, with an average near 15 ksi. The coordinates had values between 0 and 6 in.

3.1 Principal Component Analysis. The stress and coordinate data at the nodes were reduced through PCA. While the PCs of the stress show the variation of the stress through the design space, the PCs of the coordinates were used to relate stress variations to geometric variations. The reduction was performed on the data matrix, X , shown in Eq. (1) and assembled as shown in Fig. 2. Each of n rows in the matrix represented a finite element model from one sample. The columns, of which there were p , represented the nodes of the finite element model. The first quarter of the columns were stresses at the nodes and the remaining columns were coordinates at the nodes. This meant that p was equal to four times the number of nodes, m . For these tests, m was 25,000 and n was between 50 and 100. Only nodes on the surface of the finite element model were used to decrease the computation while retaining all visible node information useful in design space exploration. Common practice for the structural design of compressor blades only considers stress on the surface

$$X - \bar{x} = USV = AV \tag{1}$$

The PCA method used to decompose the data matrix is thoroughly explained by Jolliffe and Cadima [9]. This research centered the data by subtracting the column means, \bar{x} . The result of PCA on the data matrix was twofold. First, a matrix, V , containing the PCs as rows. Next, a matrix, U , and a diagonal matrix, S . When these two latter matrices were multiplied, they return a matrix, A , representing the scores of each PC for each sample as shown in Fig. 3. PCA finds the matrices A and V that satisfy Eq. (1). A reverse transform may be used to obtain the nodal stress and coordinate data, x , for a design. This is shown in Eq. (2) where a is a vector containing a single score for each PC

$$x = aV + \bar{x} \tag{2}$$

3.2 Two-Point Correlation. The two-point correlation was performed between the design variables and PC scores. The correlation for every combination of PC and design variable was obtained by Eq. (3). In this equation, d_i is a vector of design variable values for the i th design variable, and pc_j is a vector of PC scores for the j th PC. The elements in the vectors are from the samples used in

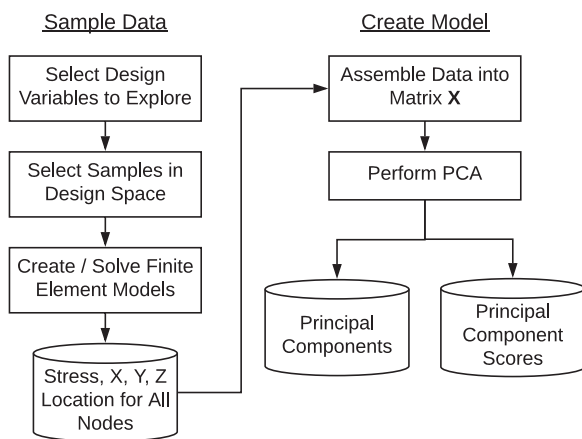


Fig. 1 Flowchart for PCA of finite element models. Rectangular boxes are processes while cylinders represent data.

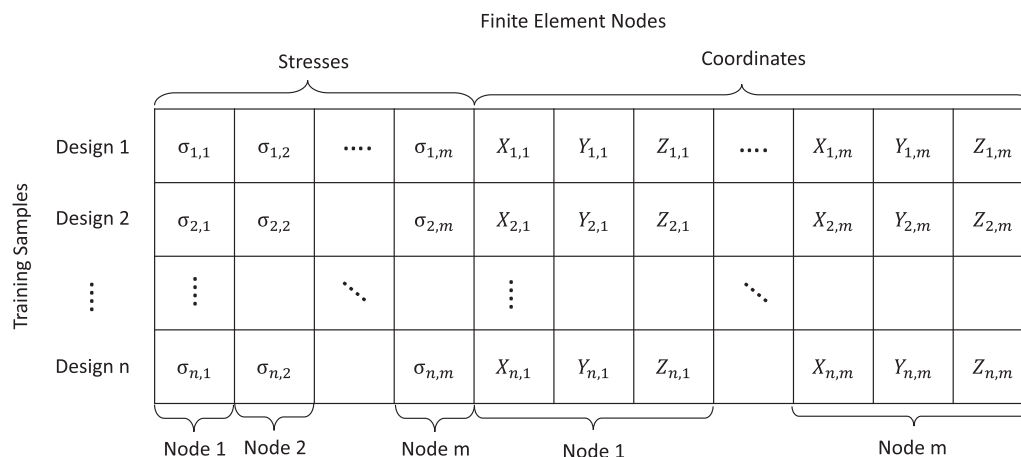


Fig. 2 The (n, p) data matrix, X . Each row was a unique finite element model sample, of which there were n . The columns, of which there were p , represented the m nodes on the finite element model.

		Principle Components			
		PC 1	PC 2	...	PC q
Training Samples	Design 1	Score 1, 1	Score 1, 2	...	Score 1, q
	Design 2	Score 2, 1	Score 2, 2		
	
	Design n	Score n, 1			Score n, q

Fig. 3 The (n, q) PC matrix, **A**. Each row was one of n samples. Each column was a PC, of which there were q .

the PCA. The value of the correlation between the i th design variable and the j th PC is $c_{i,j}$ and is used to understand the relationship between that design variable and PC. Higher correlation values indicate a better relationship between the variable and PC

$$c_{i,j} = d_i \cdot pc_j \quad (3)$$

4 Results

Several design spaces of the transonic Purdue compressor blade were tested to show how two-point correlation and stress and geometry coupling improve DSE. The first few PCs and mean stress from a design space with all nine design variables are shown in Fig. 4. The number of samples, n , was 100 in this example. The upper color scale in the figure is for the stress in Fig. 4(a) and uses the standard color palette in understanding the stress distribution for FEA post processing. Figures 4(b)–4(d) use the lower color scale in the figure to show the stress variation of the PCs. Red shows the increase of stress and blue shows decrease while white shows no variation. Each PC in the figure shows normalized stress variation obtained by dividing the variation by the maximum absolute stress variation of that PC. Each PC is normalized to its own variation so that each PC may be clearly seen and understood. All other PC figures will use this same scale. The stress contours for any design within the nine-dimensional design space may be reconstructed by a linear combination of these PCs and the mean stress. The scores associated with each design indicate the coefficient for the linear combination. The figure shows the pressure surface of the blade with the leading edge of the blade on the right. The mean stress profile among the samples, shown in Fig. 4(a), shows the highest stress of about 34,500 psi at the bottom corners of the blade and high stress in the center of the blade reaching up to about 25,000 psi. PC-1 in Fig. 4(b) shows a stress increase at the trailing and leading edge, especially at the bottom stress concentrators. It also has decreased stress in the upper right and lower left regions of the blade. PC-2 in Fig. 4(c) displays an area of large stress increase mid-way between the leading and trailing edges from the bottom to about 60% up the blade with gradual reduced variation away from this region. It also shows a decrease in stress along the leading and trailing edges. PC-3 in Fig. 4(d) shows alternating increase and decrease along the chord of the blade.

The explained variance, or percent of total variation in the data each PC describes, decreases with each consecutive PC. This is shown with a scree plot in Fig. 5 where bars are the explained variance of each PC and the dashed line is the cumulative explained variation. Figure 5 shows, with the height of the left-most bar, that PC-1 explains close to 70% of the variation in the stress contours across the nine variable design space. This is followed by 20%

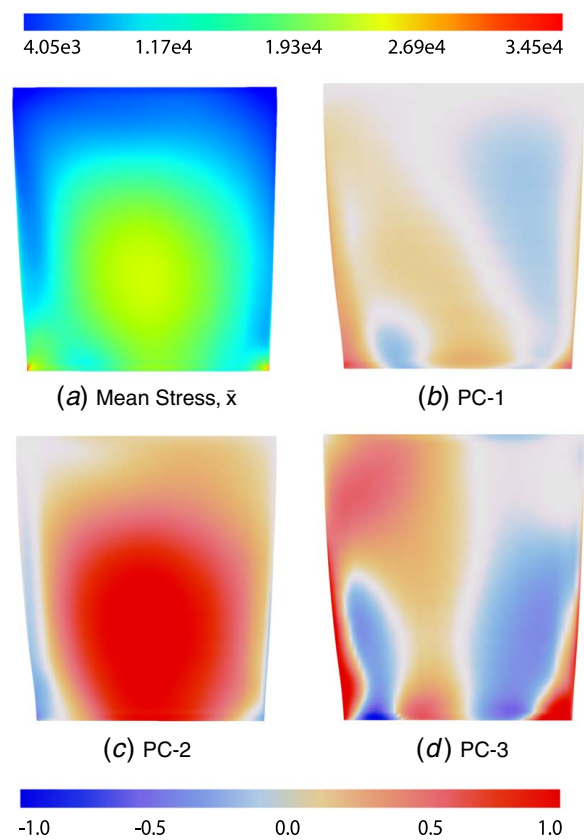


Fig. 4 Mean stress and three PCs for the design space of nine variables in Table 1. Image (a) is scaled typical to FEA, using the upper color scale with units of psi. Images (b)–(d) are PCs which represent normalized stress variation and use the lower color scale. The variations are normalized by dividing each nodes value by the maximum absolute stress variation.

with PC-2 while the others are less than 10%. The first three PCs explain about 95% of the variation among the samples.

4.1 Low-Dimensional Relationships. To use the PCs as a DSE tool, a designer must understand the relationship between the PC scores and the design variables. For design spaces with

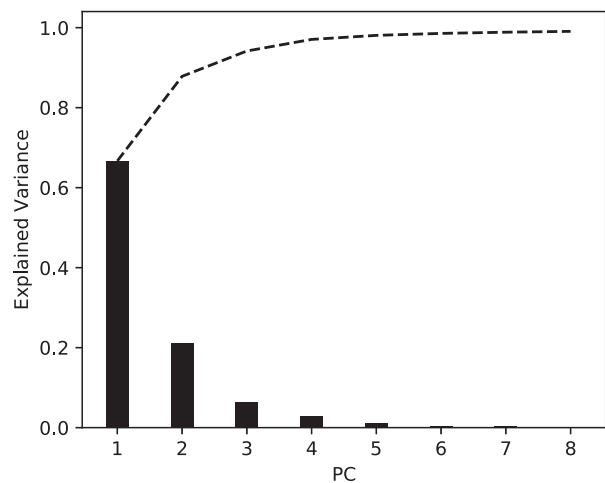


Fig. 5 Scree plot of PCs explaining 99.9% of variation in the design space of nine variables in Table 1. Bars are each PC explained variance and dashed line is cumulative.

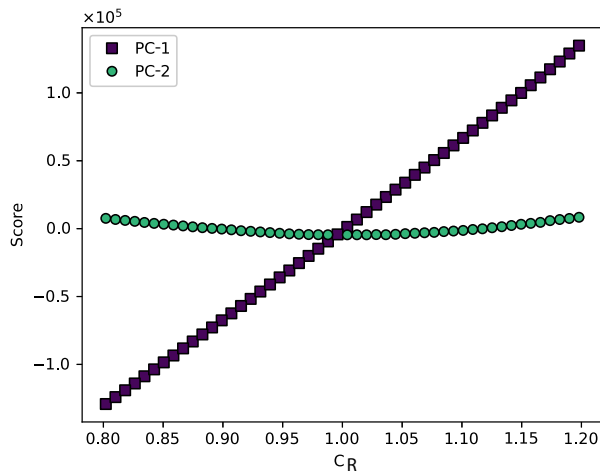


Fig. 6 Scores for all PCs explaining 99.9% of the variation across a design space of C_R

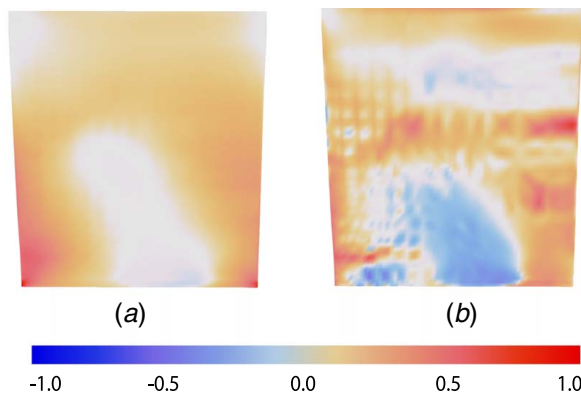


Fig. 7 First two PCs over a design space of C_R : (a) PC-1 and (b) PC-2

one variable, every sample's PC scores can be plotted against the variable as shown in Fig. 6. This shows the PC scores for the Purdue blade with a design space which only varies C_R and where the number of samples, n , was 50. As this variable increases, the score of PC-1 increases significantly. This means that the variation of stress shown in Fig. 7(a) is increased in the full stress field of the design as C_R increases. While the score of PC-2 increases in either direction as C_R moves away from a value of one, this change is much smaller than that of PC-1. The amount each PC score may vary in the design space is related to the explained variance of the PC. Over this design space, PC-1 explains most of the variation causing its score to change more than the other PCs. According to Fig. 6, as C_R moves away from a value of 1.0, the full stress profile of the blade changes in small amounts according to the variation shown with PC-2 in Fig. 7(b). Using this figure and Fig. 6, a designer can know how changing C_R increases or decreases the stress at the regions described by the PCs.

Design spaces with two variables can use three-dimensional plots to show the scores across the design space. Figure 8 is similar to Fig. 6 but extended to three dimensions to allow two design variables. Because of the interaction between design variables, the scores do not line up as nicely with respect to any one design variable as in Fig. 6. Plotting the scores against only one dimension shows more noise and obscures the effect of other variable on the score.

Figure 8(a) shows a three-dimensional plot from the α -axis. The number of samples, n , for this example was 100. This perspective shows that PC-1 decreases with increasing α and PC-2 increases

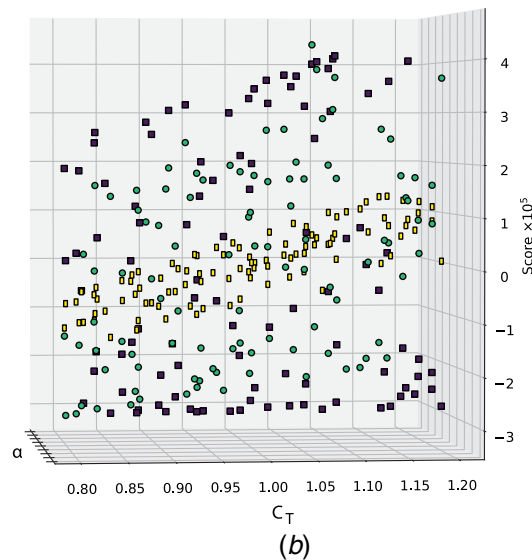
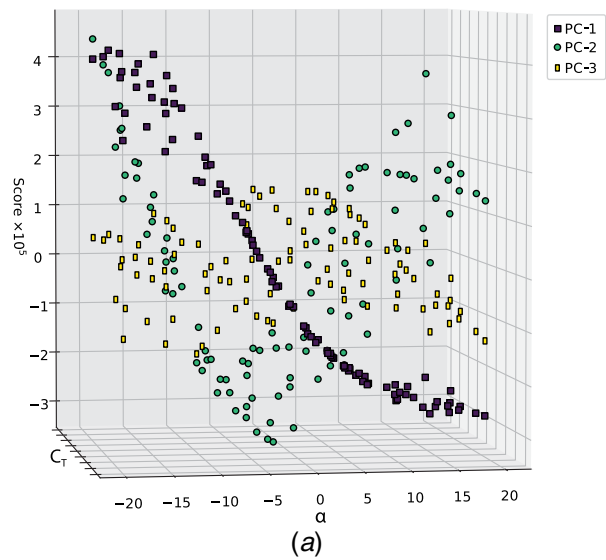


Fig. 8 Scores for the first three PCs of the variation across a design space of C_T and α : (a) scores for PCs from α axis and (b) scores for PCs from C_T axis

as α is moved away from zero. As α increases from -20 to about -5 , the contribution of PC-1 and PC-2 to the stress distribution decreases while PC-2 increases. Because PC-3 has no noticeable trend across the range of α , this design variable can likely be changed without influencing that PC. Figure 8(b) shows the plot from the view of the C_T axis. Due to the larger scatter of data in PC-1 and PC-2, the image shows that they do not correlate as much with C_T as they do with α . The scores for PC-3 exhibit a tighter grouping with respect to this variable than with α . This shows that this variable correlates more with C_T than with α . As such plots are used interactively, designers can view the plot from different perspectives to understand how PC scores change across the design space of two design variables. This can help them know which design variables to change and which regions of the design space to explore to reduce the stress at areas of the blade indicated by the PCs.

4.2 High-Dimensional Relationships. Understanding these relationships when a design space has more than two variables is more difficult. Plots similar to Figs. 6 and 8 can be created if the designer can find one or two design variables which have the

greatest influence on the PC scores. Discovering these variables is difficult and there may not be only two which clearly relate to the PC scores. This section discusses two novel methods to find the variables which most relate to the PC scores.

4.3 Two-Point Correlation. The first method applies a two-point correlation to relate the design variables to the PCs for DSE of compressor blades. Figure 9 shows correlation scores with PCs along the x -axis, and Fig. 10 shows scores with design variables along the x -axis. These figures use the same design space and training samples discussed with Fig. 4. Both figures show a correlation between the PCs and the design variables but with different groupings. It is more intuitive to find which variables relate to each PC in Fig. 9 and more intuitive to find which PCs relate to the design variable in Fig. 10. L is shown to have the largest effect on PC-1 in Fig. 9. C_R has the next largest effect upon PC-1, but its influence upon the component is much less than L . While PC-2 does have some correlation with L , it does not have a single design variable with which it correlates significantly more than the others. Figure 9 shows the correlation with L and multiple PCs, but reading and comparing the correlations of multiple PCs with L in the figure can be difficult. Figure 10 helps visualize the correlation better from this view point. This shows that as L is changed, there is a significant change in PC-2 but not as much as PC-1.

4.4 Stress and Geometry Coupling. The second method to understand the relationship between the PCs of the stress and the design variables is to consider the geometric portion of the PCs in a stress and geometry coupled PCA. Because the node locations were coupled with the node stress for PCA, the PCs contain variations not only of stress but also of geometry. However, because the magnitude of the values which represent the stress are several orders of magnitude greater than the values which represent the geometry, the PCA is weighted toward the stress. The stress portions of the PCs in the coupled analysis are nearly identical to the stress components when stress and location are segregated. The geometric portions of the coupled PCs are only the geometric variations which align with the stress variations. Figure 4 shows only the stress portion of the PCs contoured onto a blade of nominal geometry. The geometric portion of the component, however, may also be graphically displayed along with the stress contour. It is easier to understand how the design variables relate to the geometric variations because the variables directly control the geometry. These geometric portions of the PC can be used to know which design variables corresponded to the PCs.

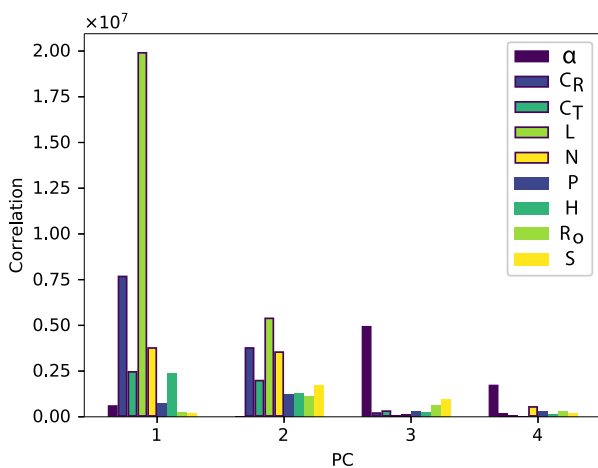


Fig. 9 Correlation scores for the first four PCs of the nine-dimensional design space. PCs are along the x -axis while the design variables are colored bars.

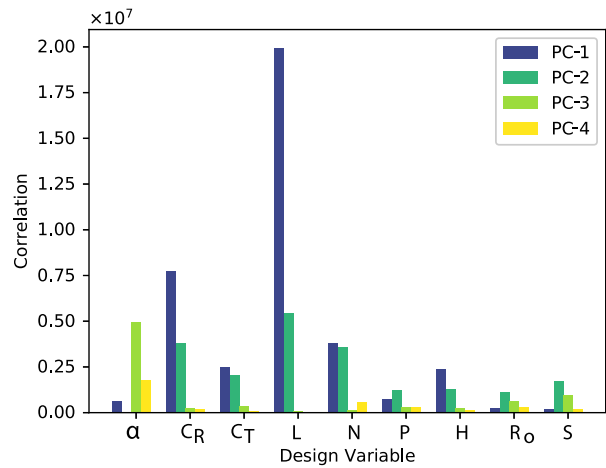


Fig. 10 Correlation scores for the first four PCs of the nine-dimensional design space. Design variables are along the x -axis while the PCs are bars.

Figure 11 displays the first four PCs of the Purdue blade with the nine-dimensional design space. The mesh is the nominal blade geometry while the solid, contoured geometry shows the geometric variation. The colors still represent the normalized stress portion of the PCs. Figure 11(a) shows that PC-1 has a geometric variation which includes mostly L . This is determined by visual inspection. The solid geometry leans back from the mesh perpendicular to the blade faces. This is how L changes the geometry. While there is geometric variation shown for PC-2 in Fig. 11(b), no single design variable from Table 1 describes the majority of this variation. PC-3 and PC-4 show twist of the solid geometry around its vertical axis. This twist is caused by a change in α . These all agree with the two-point correlations shown in Fig. 9. This method, however, also provides an indication to directionality that the two-point correlation does not. Figure 11 shows that the score of PC-1 increases with negative L . This means that as L is changed such that the blade leans back away from nominal the stress will change by an increase of the variation described by PC-1. Figure 9 only shows that L is related to PC-1, but does not show the direction the relationship. The two-point correlation method, however, also works for correlations to the design variables which do not change geometry such as N and P . The stress and geometry coupling method would not capture these correlations.

5 Discussion

This discussion shows how to use the PC-based methods to choose which design variables and regions of the design space should be explored to change the blade stress distribution. The process for using these methods is outlined as shown in Fig. 12, and then, an example is used to show each step of the process in detail. This example uses nine design variables and 100 samples. This example will show that these methods help improve the understanding of the structural design space by showing the effect of specific variables on the stress distribution of the blades.

5.1 Process. The process in Fig. 12 is based on understanding the relationship between the design variables and the PCs. First, after reviewing the stress distribution for a specific design, regions of the blade are selected based on where the designer wishes to explore how to change the stress and the resulting effects. Second, the PCs which influence stress at the desired regions are selected. Because the PCs show stress contour changes through the design space, changing the score of a given PC will change the stress as described by the PC. The explained variance, like that shown in Fig. 5, may also be used in this step. Using

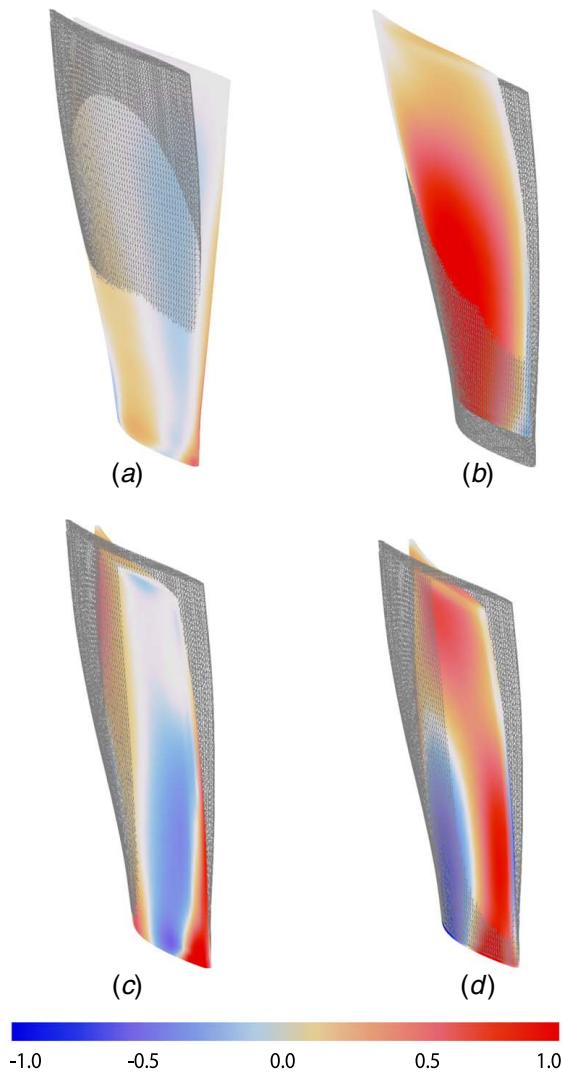


Fig. 11 First four PCs with geometric variation over the design space of all nine design variables shown in Table 1. Mesh shows nominal geometry. The displacements of the geometry are scaled in each image such that the shape of the variation is visible: (a) PC-1, (b) PC-2, (c) PC-3, and (d) PC-4.

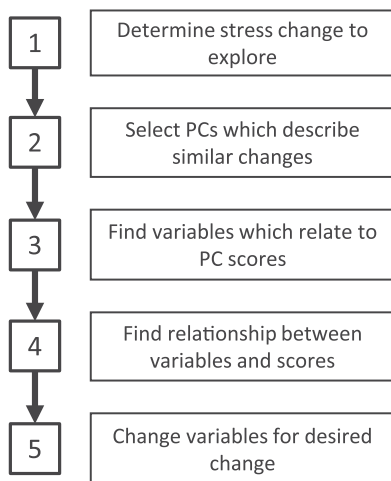


Fig. 12 Process for using the relationship between PCs and design variables to change stress

PCs with higher explained variance will have the greatest influence on the stress. Third, the design variables which relate to the selected PCs are chosen. This is done by using the correlations or stress and geometry coupling methods described in Secs. 3 and 4. Fourth, the relationship between the selected design variables and PCs is discovered. The correlations and coupling, from the third step, indicate a relationship between design variables and changes of stress described by the PC. Plots, such as those described in Figs. 6 and 8, may be used to understand how they are related. Lastly, once an understanding of how the design variable changes the stress is determined, the design change is made to change the stress as desired in step one. The effects of this change are observed to improve the understanding of how the design variables change the stress and how stress changes at specific regions affects stress at other regions.

5.2 Example. An example of the process is given using Fig. 13. The von Mises stress for a nominal design is shown in Fig. 13(a). Most of the stress at this design is below 10,000 psi; however, the stress at the bottom corners is greater than 30,000 psi. For step one in Fig. 12, it is desired to explore how to reduce the stress at the bottom corners to below 20,000 psi and the resulting effects of those changes. Second, the PCs from the design space in Fig. 4 are used. Figure 4(b) shows that an increase in the score of PC-1 increases the stress at the bottom corners. Figure 4(c) shows that an increase in the score of PC-2 decreases the stress at the bottom corners. Figure 4(d) shows that an increase in the score of PC-3 increases the stress at the bottom corners, but this PC also shows a decrease in stress very near the bottom left corner for the same increase of score. From these PCs, it is determined that the score of PC-1 should decrease and the score of PC-2 should increase. Figure 5 shows that PC-1 and PC-2 explain about 70% and 20%, respectively, of the stress variation through the design space. This means that using variables which relate to the scores PC-1 is likely to achieve greater stress change than those variables which relate to the scores of PC-2.

Step three requires finding design parameters which relate to PC-1 and PC-2. Figure 9 shows that PC-1 is mostly influenced by L . Therefore, L should be used to change the stress. Figure 9 also shows that there is no design variable which correlates with PC-2 significantly more than other variables. Therefore, no additional variables will be used for this iteration of the process. Similar conclusions may also have been obtained by using the stress and geometry coupling in Fig. 11 instead of the correlations in Fig. 9. Step four finds the relationship between L and all PCs which relate to L . Figure 10 shows that the only significant correlations with L are PC-1 and PC-2. Figure 14 shows the scores of these PCs across L . A value of 0.15 is chosen for L because the score of PC-1 is at a minimum and the score of PC-2 is close to a maximum. The fifth step is to change the design and check the solution. Figure 13(b) shows that the stress at the bottom corners was decreased from greater than 30,000 psi to less than 10,000 psi. Through this process, the designer learned that L may be used to change the stress at the bottom corners of the blade. The effect of changing L from the nominal design also increased the stress to greater than 30,000 psi at the center of the blade.

To gain more precise influence and understanding of the stress, more iterations of this process may be performed which explore using design variables not selected in previous iterations. Exploring L successfully decreased the stress at the bottom corners. However, because an increase in the score of PC-2 also increased the stress at the center of the blade from less than 10,000 psi to greater than 30,000 psi, it may be desired to further explore the design space. The process is begun again by choosing to explore how to reduce stress at the center of the blade and the resulting effects. PC-2, from Fig. 4(c), is chosen for step two because it describes a change in stress at the center of the blade. It is desired to decrease the score of PC-2 without a significant increase in the score of PC-1 in Fig. 4(b). Steps three and four are more difficult with subsequent

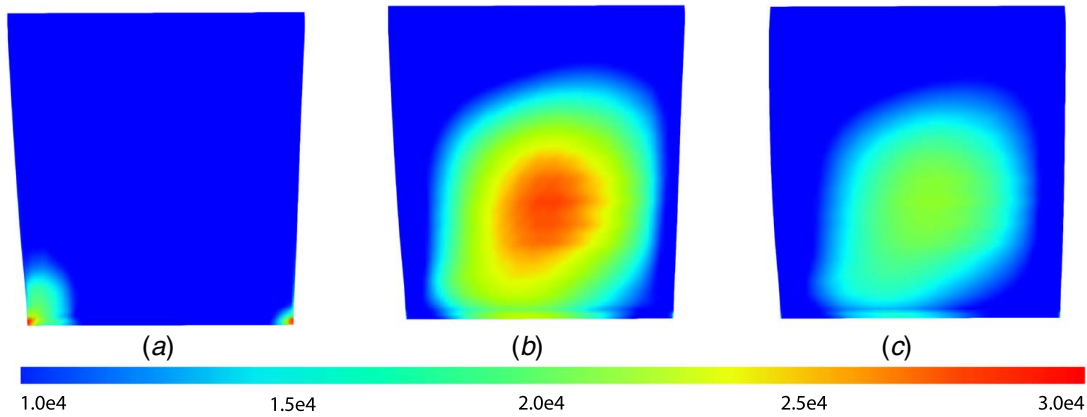


Fig. 13 DSE of the Purdue compressor blade with the nine-dimensional design space using the design process described in Fig. 12. The von Mises stress is shown for three designs. Each uses the given color bar with units of psi. Any stress above 30,000 psi is shown as red and any stress below 10,000 psi is shown as blue: (a) nominal design, (b) increased L , and (c) increased L , and decreased N .

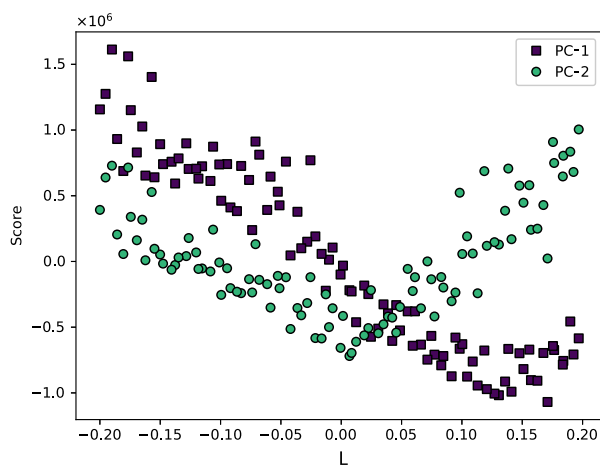


Fig. 14 The scores for the first two PCs with respect to L in the nine-dimensional design space

iterations. Changing the stress is more challenging when the variables which correlate to a PC have close to the same or greater correlation with other PCs as well. For step three, Fig. 9 shows that any variable that has significant correlation with PC-2 also has significant correlation with PC-1. This suggests that multiple variables need to be investigated.

The next variable for this example will be C_R . Figure 10 shows that only PC-1 and PC-2 have significant correlations with C_R . For step four, Fig. 15 shows how the PC scores vary across C_R and L . This plot shows a surface which is fitted to the samples across these variables with a black line indicating where L equals 0.15 on the surface. Because L was set to 0.15, any change in C_R should result in the scores following these lines. Figure 15 shows that changing C_R at this value of L will not result in a significant change of score for PC-1 or PC-2. Using C_R will not likely help achieve a reduction in stress at the middle of the blade. Next, the process returns to step three and N is chosen as the design variable. In step four, Fig. 16 shows that a decrease of N , where L equals 0.15, will decrease the score of both PC-1 and PC-2. This will decrease the stress center of the blade due to PC-2. Decreasing the score of PC-2 would also increase stress at the bottom edges, but because the score PC-1 also decreases, this effect should be canceled out. For the fifth step in this iteration of the design process, N is changed to a value of 1.1×10^6 . This value is chosen because Fig. 16 indicates that the scores PC-2 and PC-1 are close to their minimums for L equals 0.15. Figure 13(c) shows that with a new

design of increased L and decreased N , stress was reduced to about 20,000 in the center of the blade without also increasing the stress above 10,000 at the bottom corners. This example has shown how using the methods which discover the relationship between the PCs and the design variables can assist in exploring the design space, can increase the understanding of the design space and can be used to achieve design objectives.

5.3 Real-Time Design Exploration. The methods applied and developed in this research help designers know which variables and regions of the design space should be explored for desired structural results. As shown through the example, the process may take a few iterations to find a satisfactory design. These methods could be applied with real-time DSE as done by Bunnell et al. [1] to further improve DSE. The exercise shown in Fig. 13 provides guidance to indicate which variables to explore in large design spaces. Fine tuning a design around a given region of the design space may require many iterations. A desired result can be quickly achieved by using real-time DSE to quickly search regions of the design space indicated by the methods described in this paper.

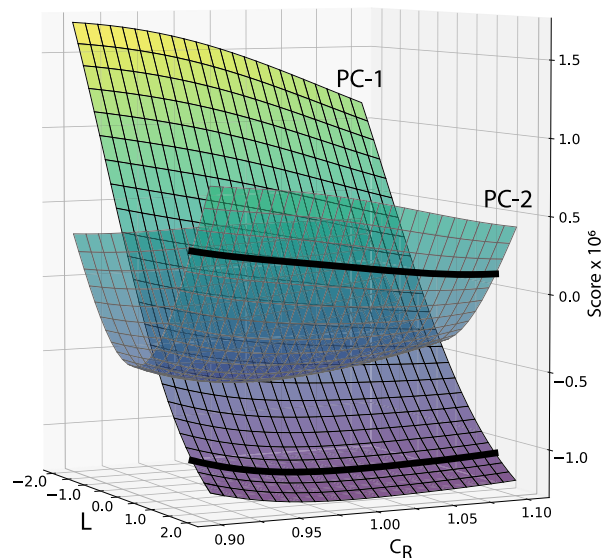


Fig. 15 The scores for the first two PCs with respect to C_R in the nine-dimensional design space

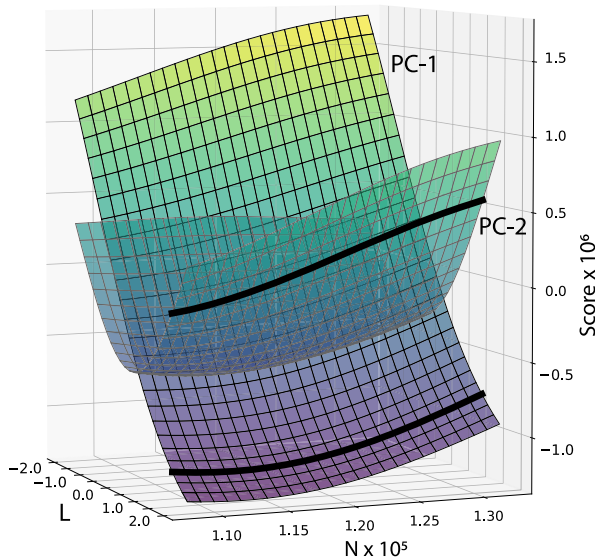


Fig. 16 The scores for the first two PCs with respect to N in the nine-dimensional design space

6 Conclusion

This research used PCA to reduce structural results from FEA into PCs describing the structural variation of compressor blades across design spaces. Two methods were applied which related the design variables to PCs. These methods allowed the PCs to be used as a DSE tool by indicating which variables and regions of the design space should be explored to changes at specific regions of the blade and the effect of those changes. The two-point correlation, an existing method, was applied to the design variables and the PC scores. This correlation indicated the design variables which had the greatest influence upon each PC. It was shown how these correlations can guide DSE and find the design variables which need to be changed to best influence the structural results. The geometric coupling method was developed and applied to relate variations in stress directly to variations in the geometry of the compressor. Like the correlation method, this method indicated which design variables should be changed to achieve desired structural results. The geometric coupling method also provided an indication to the direction the design variables should be changed. These methods performed well in high-dimensional design spaces that are common in turbomachinery compressor blade design.

The methods presented in this paper should be used when performing structural design exploration based on finite element analysis. A DOE is used to select a set of samples distributed globally through the design space. This research found useful exploration with relatively few samples, i.e., 100 when using nine design variables. FEA is then performed for the sampled designs to obtain the results with the structural response and coordinates which will be explored. The results from FEA need to be represented on a grid which is common among all of the samples in order to keep the nodes in the same location relative to the geometry. This research accomplished a common grid representation with mesh morphing [1]. PCA should then be performed as described in Sec. 3. The PCs describe the variation of the structural response contours throughout the design spaces. These are used to understand how the structural response may be changed with the used design variables. PCs which describe more variance allow more change in the structural results. This research found the first two or three PCs explained more than 95% of the variation with tests that included nine design variables. If a desired change is not obtainable, other variables should be used. This will change the PCs and the amount of variation each one describes. The scores of the PCs can then be related to the design variables to understand the effect of the variables on the structural

response. Either two-point correlation or stress and geometry coupling may be used to relate the designs. Once the understanding between variables and structural response is known, a new design may be wisely selected and analyzed.

As design spaces for compressor blades become more complex, better methods are needed to more fully understand them. This research developed and applied methods that use PCA of the design space to show variations which are not easily understood otherwise. These methods provide guides to change the design variables by which the stress is better understood and controlled. The improved understanding provided by these methods can help meet design objectives and decrease the computation cost needed to achieve them.

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