ABSTRACT

Collaborative products are created by combining components from two or more products to result in an additional product that performs previously unattainable tasks. The resulting reduction in cost, weight, and size of a set of products needed to perform a set of functions makes collaborative products useful in the developing world. In this paper, a method for designing a set of products for optimal individual and collaborative performance is introduced. This is accomplished by: (i) characterizing the collaborative design space of the product set and collaborative product, (ii) defining areas of acceptable Pareto offset, (iii) identifying the combinations of designs that fall within the defined areas of acceptable Pareto offset for each product, and (iv) selecting the optimal set of product designs. An example is provided to illustrate this method and demonstrate its usefulness in designing collaborative products for both the developed and developing world. We conclude that the presented method is a novel, and useful, approach for balancing the inherent trade-offs between the performance of collaborative products and the product sets used to create them.

Keywords: Collaborative Products, Multiobjective Optimization, Modular Product Design, Engineering-Based Poverty Alleviation.

1 Introduction and Background

Poverty is a growing concern throughout the world. As reported by the World Bank in 2005, those in extreme poverty (1.4 billion people) live on less than $1.25 a day [1]. Groups working to assist these individuals to escape poverty have noted that one of the major challenges these people face is the inability to generate sufficient income [1]. As such, in recent years an engineering focus has developed that emphasizes sustainability in the developing world by designing products that generate income for the user [2, 3]. Through the development and distribution of such products, poverty-stricken individuals are able to benefit from improved health and education by increasing their income [2]. Even though more than 12 million people have been brought out of poverty with such products, there still exists a large concern for others due to the financial risks that exist when purchasing such products [2,3]. As a result, if financial risks can be lowered, the impact of these products will expand to more people who previously considered these products to be unaffordable. The goal of the method introduced herein is to design products that not only generate income, but appeal to a greater number of individuals because of their affordability.

There have been few areas within product design research aimed at bringing individuals out of extreme poverty. Some of these include the design of reconfigurable products and modular products [4–6]. Within this paper, is a combination of reconfigurable and modular product design approaches identified as collaborative products [6].
laborative products are created when physical components from two or more products are brought together to form another product capable of performing additional tasks [6].

Collaborative products have the potential to significantly influence the impact that income-generating products can have on poverty alleviation efforts by reducing the cost of a set of products capable of performing a specified set of tasks. This is accomplished by increasing the task-per-cost ratio of a set of products [6] so as to reduce the number of products needed to perform a set of tasks. It is this ability to perform a set of tasks with fewer products that effectively lowers the financial risk for the user and increases their likelihood of purchasing these products. There exists a method for designing collaborative products, consisting of an eight step process [6]. The steps of this method are:

1. Select Product Category
2. Perform Extensive Product Search
3. Decompose Products into Primary Components
4. Identify Optimized Product Sets
5. Select Product Set
6. Add Missing Components
7. Identify Interfaces
8. Complete Detailed Product Design

Recognizing the inherent trade-offs and compromises that must be explored in order to determine the design of each product in the collaborative product set, the method presented in this paper builds upon the method presented in Morrise et al [6] by expanding Step 8 above to incorporate optimization methods. This is useful to the designer in selecting the optimal designs for the set of products used to create a collaborative product. This process is further explained in Sec. 2.

It is noted that modular product design is essential in the design of collaborative products as it involves the joining together of multiple products. In the literature this type of modularity is known as Type II modularity, defined as the design of interfaces with modules that can only be attached to other specific modules through a unique interface [7].

The method presented in this paper for designing collaborative products also involves many changing and competing needs that must be addressed to successfully design a product. One way to meet these demands and resolve the competing nature of both present and future needs of a set of products is through multiobjective optimization [8–10]. This technique serves as a fundamental foundation to the design method presented in this paper. Multiobjective optimization characterizes the trade-offs between design objectives by identifying a Pareto frontier or a set of non-dominated optimal solutions. These Pareto solutions are of importance because they show that design objectives have been improved to their full potential without sacrificing the performance of objectives in other areas [4,8–10].

A set of optimal solutions belonging to the Pareto frontier can be found through the following generic multiobjective optimization problem presented as Problem 1 (P1):

$$\min_x \{ \mu_1(x,p), \mu_2(x,p), \ldots, \mu_{n_p}(x,p) \} \quad (n_p \geq 2) \quad (1)$$

subject to:

$$g_q(x,p) \leq 0 \quad \forall q \in \{1, \ldots, n_g\} \quad (2)$$

$$h_k(x,p) = 0 \quad \forall k \in \{1, \ldots, n_h\} \quad (3)$$

$$x_{jl} \leq x_j \leq x_{ju} \quad \forall j \in \{1, \ldots, n_x\} \quad (4)$$

where $\mu_i$ denotes the $i$-th generic design objective to be minimized (i.e., cost or size of a product); $x$ is a vector of design variables that define the design of a product (i.e., length, width, height); $p$ is a vector of design parameters (i.e., material yield strength, modulus of elasticity) that will be treated as constants in the optimization; $x_u$ and $x_l$ define the upper and lower bounds of the $j$-th design variable; $g$ is a set of inequality constraints; and $h$ is a set of equality constraints. Note that the objectives and constraints are functions of both $x$ and $p$, and that the objectives will be minimized by changing the values of $x$.

Aside from the developing world, collaborative products can also apply in the developed world. Many individuals within the United States suffer from poverty, living in small dwellings with limited storage space. Money is also limited for these individuals, and collaborative products are a way to help maximize space while providing a set of product functions that are extremely affordable [6]. Other identified areas that could benefit from collaborative products may include payload conscious industries such as aerospace and backpacking [6].

The remainder of this paper is organized as follows: The theory for designing products for optimal individual and collaborative performance is found in Sec. 2. In Sec. 3, the design of a collaborative brick press demonstrates implementation of the presented method, followed by concluding remarks Sec. 4.

2 Method of Designing Products for Optimal Collaborative Performance

This section presents a method for designing a set of products for optimal collaborative performance. In selecting the design of a product that will to be reconfigured for use in a collaborative product, the impact of design changes on the performance of the individual and collaborative product must be considered. In the context of multiobjective optimization, product performance is typically correlated to design objective values for each product.
Recall that points along the Pareto frontier (graphically illustrated in Figure 2) represent the best possible trade-offs between the selected design objectives of each product. Although a design is located on the Pareto frontier of an individual product, the corresponding performance of the collaborative product, and the other products in the set, are not guaranteed to be Pareto in each product’s objective space. As such, the collaborative performance of a product is inversely correlated to the measured offset of a design from the corresponding Pareto frontier of that product. Building on this definition, the optimal collaborative performance of a product set is therefore obtained by maximizing the collaborative performance of each product simultaneously.

Recognizing the inherent trade-offs and compromises in collaborative performance that must be explored, the purpose of the method presented in this section is to implement an optimization-based approach to mitigating these trade-offs. Figure 2 graphically represents the intent of balancing these trade-offs using the method presented in this section for two products that are combined to create a third product. Although the presented method is not limited to the simple case presented in Figure 2, a limited number of products are used for simplicity of visualization purposes. From the figure it can be observed that the presented optimization routines select designs for each product that fall within identified offset areas within each objective space. Figure 1 illustrates the four-step optimization-based method presented in this section. A discussion of each step is now provided.

### 2.1 Step 1: Characterize the Collaborative Design Space

Assuming that Steps 1-7 of the method presented in Morrise et al [6] have been completed (see Figure 1), this method begins with a knowledge of the product set and corresponding collaborative product that is desired. In order to enable the use of optimization methods to explore possible design solutions, objectives for each of the products in the set and the collaborative product are identified, and models of these objectives are created that incorporate the intended product interfaces. Using the developed models, the design space of each product is determined by a multiobjective optimization problem similar to \((P1)\).

In order to define each product, and identify the variables that couple the design of each product in the set to the collaborative product, the design variables for each product are divided into three groups: interface variables \((x_I)\), collaborative variables \((x_C)\) and adjustable \((x_A)\) variables. The interface or platform variables are shared throughout the product set and define the connecting interface between each product. The collaborative variables are those connected to the elements of a product that are used to create the collaborative product. The adjustable variables are those connected to the elements of a product that are unique to each product in the product set.

The characterization of the multiobjective design space for the \(i\)-th product in the set, and the collaborative product \((i = n_p + 1)\), in terms of identifying the corresponding Pareto frontier (See Figure 2) is presented as Problem 2 \((P2)\):

\[
\min_{\hat{x}^{(i)}} \left\{ \mu_1^{(i)}(\hat{x}^{(i)}, p^{(i)}), \ldots, \mu_{n_{\mu}}^{(i)}(\hat{x}^{(i)}, p^{(i)}) \right\} \quad (n_{\mu}^{(i)} \geq 2) \quad (5)
\]

subject to:

\[
g^{(i)}_{q^{(i)}}(\hat{x}^{(i)}, p^{(i)}) \leq 0 \quad \forall q^{(i)} \in \{1, \ldots, n_g^{(i)}\} \quad (6)
\]

\[
h^{(i)}_{k^{(i)}}(\hat{x}^{(i)}, p^{(i)}) = 0 \quad \forall k^{(i)} \in \{1, \ldots, n_h^{(i)}\} \quad (7)
\]

\[
\hat{x}^{(i)}_j \leq \bar{x}^{(i)}_j \leq \hat{x}^{(i)}_j \quad \forall j \in \{1, \ldots, n_{\bar{x}}^{(i)}\} \quad (8)
\]

\[
\hat{x}^{(i)} = \left[ x_{1,1}^{(i)}, x_{1,2}^{(i)}, \ldots, x_{1,n_{x_I}}^{(i)}, x_{C,1}^{(i)}, x_{C,2}^{(i)}, \ldots, x_{C,n_{x_C}}^{(i)}, x_{A,1}^{(i)}, x_{A,2}^{(i)}, \ldots, x_{A,n_{x_A}}^{(i)} \right] \quad (9)
\]
where \( \hat{x}^{(i)} \) is a vector of design variables containing the interface \((x_I)\), collaborative \((x_C)\), and adjustable \((x_A)\) variables for the \( i \)-th product in the set. The Pareto frontier of each product is obtained by evaluating \((P2)\) \( \forall i \in \{1, 2, \ldots, n_p + 1\} \).

It should be noted that in Equation 9, the collaborative variables of the collaborative product \((i = n_p + 1)\) encompass all collaborative variables from the identified product set. This coupling of the product set to the collaborative product design space is important since it illustrates to the designer the current collaborative nature of the product set.

### 2.2 Step 2: Define the Areas of Acceptable Pareto Offset

In looking at the formulation of \((P2)\), the resulting Pareto frontier for each product represents the best possible solutions for each of the products without considering the interaction between each product. It is noted that as the number of products being combined increases, the less likely it will be that the designs capable of creating a collaborative product all fall on the Pareto frontier of the corresponding product. This is because the number of objectives and constraints to be satisfied, along with the complexities of the interactions between the products, increases with each additional product. As more interactions and trade-offs become apparent, the harder it is to meet all of the demands between products. In order to facilitate the selection of designs that will minimize the offset from these Pareto frontiers, the next step in the method is to use these Pareto frontiers to define areas of acceptable Pareto offset for each product (see Figure 2).

This process is carried out by defining a single offset value \((\beta)\) for each product that will limit subsequent optimization routines to only consider designs with offsets from the Pareto frontier that are less than \(\beta\). In the case of a two dimensional model, the values of \(\beta\) would be equivalent to defining a circle of radius \(\beta\) around each identified Pareto point from Step 1. In n-dimensional cases, the value of \(\beta\) represents the maximum allowable length of an n-dimensional vector between a design option and the closest Pareto point. This value is determined by the designer based upon the extent that he or she wishes to limit the search space and focus optimization searches to the identified offset areas.

### 2.3 Step 3: Identify the Designs that Collaboratively Fall within the Areas of Acceptable Pareto Offset

After the offset areas have been determined, the combinations of designs are identified for each product that falls on the Pareto frontier of a design space consisting of the predicted Pareto offsets as the axis of the space. In the case illustrated in Figure 2, these offset points would represent a three dimensional Pareto surface consisting of points from the offset area of each product. The offset space Pareto frontier is determined by a multiobjective problem statement presented as Problem 3 \((P3)\):

\[
\begin{align*}
\min_{\hat{x}} \left\{ O^{(1)}, O^{(2)}, \ldots, O^{(n_p + 1)} \right\} \\
\text{subject to Equations 6–9 and:} \\
O^{(i)}_{q(i)} \leq \beta \quad \forall q^{(i)} \in \{1, \ldots, n_p^{(i)}\}
\end{align*}
\] (10)

subject to Equations 6–9 and:

\[
O^{(i)}_{q(i)} \leq \beta \quad \forall q^{(i)} \in \{1, \ldots, n_p^{(i)}\}
\] (11)
where \( O^{(i)} \) is the n-dimensional offset length of a design of the \( i \)-th product from the corresponding Pareto frontier of that product.

The Pareto surface is constructed by adjusting the interface, collaborative, and adjustable variables. The interface and collaborative variables are shared between the optimized products and the collaborative product, while the adjustable variables are unique to each optimized product, but shared with the collaborative product. It should be noted that in cases were there are no more than two products being combined to create a collaborative product, the result of \( (P3) \) is a Pareto surface. For product sets greater than two, the graphical representation of this offset space can no longer be provided for all products simultaneously.

### 2.4 Step 4: Identify/Select the Optimal Product Design

Since the goal of the method is to select the optimal design of each product while balancing the trade-offs required to create the collaborative product, this final step of the method uses the results of \( (P3) \) to select a single set of product designs. Under ideal circumstances, the selected designs are represented by a single Pareto point on the Pareto frontier of each product (i.e., the offset of each product is zero). One method of accomplishing this selection is through the use of an aggregate objective function \( (J) \) that represents the preferences and needs of the designer. If an aggregate objective function is used, one way of reducing the computation expenses related to the optimization problem evaluations, would be to replace Equation 10 with an equation of the form of Equation 12.

\[
\min \, J(O^{(1)}, O^{(2)}, \ldots, O^{(n+1)})
\]

### 3 Case Study: Collaborative Brick Press Design

This section demonstrates the implementation of the method presented in Sec. 2 through the design of a collaborative brick press. The concept for a collaborative brick press has been provided by Morrise et al [6]. This design collaboratively uses the following six basic products to create the brick press: shovel, hoe, rake, water transportation roller, water pump, and a small cook stove. It is assumed these are potential products that a person living in poverty would be interested in purchasing as a way to improve their life situation. The ability to combine them together into an additional product would give individuals the potential to maximize their use and potentially increase their likelihood of purchasing these products. It should be noted that the intent of this example is not to show the feasibility and necessary logistics of implementing the collaborative brick press developed herein. Rather, the intent is to demonstrate the effectiveness of the method presented in Sec. 2 in identifying the optimal designs of a given collaborative product set. The case study is useful in illustrating this method due to the following: (i) Solves a challenging engineering design problem, (ii) Shows the use of complex interfaces between products and how they are addressed, (iii) Incorporates the use of actual products used or found in developing countries, and (iv) Demonstrates the use of a multiobjective optimization problem to deal with competing objectives from each product. Figures 3 and 4 illustrate the conceptual design of each product and the interfaces required to create the collaborative brick press.

![Figure 3. Decomposition of each product in the identified product set to create a brick press.](image-url)
Figure 4. Illustration of the recombination of the components from the product set in Figure 3.

mize, $\downarrow = \text{minimize}$) that were selected to characterize the performance of each product. Definitions of the objectives presented in Table 1 are as follows: (i) for the shovel, rake, and hoe the objective $\mu_1$ represents the maximum bending stress in the product’s handle; (ii) for the water roller and brick press, $\mu_1$ represents the maximum bending, shear, and buckling stress that each product could experience; (iii) for the cook stove, $\mu_1$ represents the available area for cooking food; (iv) for the water pump, $\mu_1$ represents the rate at which the pump can pump water; and (v) the objective $\mu_2$ represents the cost to purchase each product. For detailed descriptions of the models developed for each product, please contact the authors.

From the models and their corresponding functions, design variables, and design objectives we are able to construct a multiojective optimization problem of the form of $(P2)$ in Section 2.1. From this optimization problem, the design spaces for each product are defined with their corresponding Pareto frontiers (See Figure 5).

### 3.2 Case Study Step 2: Define the Areas of Acceptable Pareto Offset

In step 2 of the proposed method, the areas of acceptable Pareto offset were defined. Since we have only two objectives for each product in the product set and collaborative product, the values of $\beta$ are equivalent to defining a circle of radius $\beta$ around each identified Pareto point from Step 1. For these two-dimensional cases, the value of $\beta$ represents the maximum allowable length of a two-dimensional vector between a design option and the closest Pareto point. For our example, the $\beta$ offset values were defined as shown in Table 2 for each product.

<table>
<thead>
<tr>
<th>Table 1. Summary of the objectives that were selected for each product in the product set and collaborative product</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\uparrow / \downarrow$</td>
</tr>
<tr>
<td>Shovel</td>
</tr>
<tr>
<td>Rake</td>
</tr>
<tr>
<td>Hoe</td>
</tr>
<tr>
<td>Water Roller</td>
</tr>
<tr>
<td>Cook Stove</td>
</tr>
<tr>
<td>Water Pump</td>
</tr>
<tr>
<td>Brick Press</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 2. Defined Beta values for the normalized objectives of each product</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$ Value</td>
</tr>
<tr>
<td>Shovel</td>
</tr>
<tr>
<td>Rake</td>
</tr>
<tr>
<td>Hoe</td>
</tr>
<tr>
<td>Water Roller</td>
</tr>
<tr>
<td>Cook Stove</td>
</tr>
<tr>
<td>Water Pump</td>
</tr>
<tr>
<td>Brick Press</td>
</tr>
</tbody>
</table>
Figure 5. Graphical illustration of the Pareto frontiers for each product obtained through Step 1 of the method, and the optimal collaborative design of each product identified in Step 4 of the method.

Copyright © 2012 by ASME
3.3 Case Study Step 3: Identify the Designs that Collaboratively Fall within the Areas of Acceptable Pareto Offset

Once the offset areas were defined, the combinations of designs that fall in each offset area were identified using a multiobjective problem statement of the form of (P3) (see Section 2.3). Due to the number of products involved in this case study, a graphical representation of the results of evaluating this formulation is not meaningfully possible, and are therefore not provided.

3.4 Case Study Step 4: Identify/Select the Optimal Product Design

As was mentioned in Section 2.4, an aggregate objective function was used to select the optimal combination of product designs. For this example we used a weighted sum of O with all weights equal to one except for the brick press which was equal to 10. The weights were selected with the goal of minimizing the offset of the collaborative product (brick press) from the corresponding Pareto frontier. The resulting design selection using these weights is illustrated in Figure 5.

From the results presented in Figure 5 it can be observed that the identified design for each product, except the shovel, is located on the Pareto frontier of the corresponding product objective space. Although the selected aggregate objective function and weights were successful in identifying designs on or near the Pareto frontier of each product, the majority of these designs are located near the boundaries of the Pareto frontiers. If solutions are more desirable in a particular region of the identified Pareto frontiers, additional constraints or alternative aggregate objective functions would need to be explored.

4 Concluding Remarks

This paper has presented a method for designing products for optimal collaborative performance with application to engineering-based poverty alleviation. The primary result of this method is the ability to optimize the collaborative performance of a set of products while dealing with the various, and often complex, performance interactions between the product and the collaborative product. As described in the introduction, the task-per-cost ratio can be observed to more fully understand the potential impact a collaborative product may have on alleviating poverty. The method presented in this paper is an optimization-based strategy for selecting designs of a given collaborative product set. The ability of this method to optimize based on objectives like cost and task performance, enables the task-per-cost ratio of the product set to increase. As such, the resulting collaborative product would have a higher potential impact and application within the developing world. To illustrate application of this method, a collaborative brick press created by combining a shovel, hoe, rake, water transportation roller, water pump, and a small cook stove was provided.

From the case study, and the presented results, the authors conclude that the presented method provides an effective tool for designing products for optimal collaborative performance. The potential that collaborative products can have on the impact of income-generating products on poverty alleviation by reducing the cost, weight, and size of a set of products was presented as motivation for this work. Future work on this topic will explore the following: (i) Inclusion of design objectives and constraints that will ensure that the identified product designs embody these goals of reducing the cost, weight, and size of a set of products; (ii) Further research in the correlation of the task-per-cost ratio to the impact and implementation of a collaborative product; and (iii) Explore additional indicators, such as income generation-to-cost ratio, to better understand the impact that collaborative products will have on poverty alleviation.

Acknowledgements

We would like to recognize the National Science Foundation Grant CAREER-0954580 for funding this research.

REFERENCES